EFFECTS OF HEAT-TRANSFER ANISOTROPY IN CONVECTIVE STREAMS OF LIQUIDS AND GASES

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A study is made concerning the anisotropy of thermal conductivity in a medium in a state of strain and concerning the effect of this anisotropy on the heat-transfer characteristics during forced convection.

The anisotropy of thermal conductivity in convective streams and in strained elastic media, as revealed by experiments, has confronted researchers with the problem of how to analyze mechanical and thermal phenomena interdependently in a format more general than it is done in thermodynamics of irreversible processes. The anisotropy of thermal conductivity due to convection in a stream of fluid or due to a state of strain in an elastic medium will be analyzed here, taking into account the effect of shear anisotropy of thermal conductivity on the anisotropy of the heat-transfer coefficient during forced laminar flow of a fluid through channels.

The transfer phenomena in a moving medium are described by the equations representing the conservation laws with regard to mass, energy, momentum, and moment of momentum. For a medium which moves symmetrically ($\bar{\bar{\pi}} = \tilde{\bar{\bar{\pi}}}$), the differential form of these equations is

$$\dot{\rho} = -\rho \operatorname{div} \dot{\vec{x}}, \quad \rho \ddot{\vec{x}} = \rho \vec{f} + \operatorname{div} \bar{\bar{\pi}}, \tag{1}$$

$$\rho \dot{u} = -\operatorname{div} \vec{q} + \rho I_q + \operatorname{tr} \{\bar{\bar{\pi}} : \bar{\bar{d}}\}, \tag{2}$$

with $\bar{\bar{d}}$ denoting the strain-rate tensor

$$\bar{\bar{d}} = \frac{1}{2} (\bar{\bar{z}} + \tilde{\bar{\bar{z}}}), \quad \bar{\bar{z}} = \nabla \dot{x}; \tag{3}$$

The thermomechanical determining equations must satisfy simultaneously: 1) the principle that the properties of materials do not depend on the system of their measurement; and 2) the principle of increase of entropy or the Clausius–Duhem inequality, which imposes stringent restrictions on the determining equations and which can be stated in differential form as

$$\rho \dot{s} \geqslant \frac{\operatorname{div} \vec{q}}{T} + \frac{\rho I_q}{T} - \frac{1}{T^2} \vec{q} \cdot \operatorname{grad} T. \tag{4}$$

In addition to these basic assumptions, Truesdell has formulated the simultaneity principle [1]: "a quantity which appears as an independent variable in one determining equation will appear in all equations, provided that this will not violate the laws of physics or the conditions of invariance." This principle is very important, it expresses the interrelation between transfer phenomena. The simultaneity principle does not contradict the classical description of the laws of momentum transfer, energy transfer, and mass transfer. It can be shown, for instance, that the number of variables in the determining equations for many materials slightly off the equilibrium state becomes lower and that the transfer of individual substances is described by separate variables.

On the basis of the simultaneity principle, the determining equations for a fluid with mechanical and thermal fields can be written as follows:

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$$T = \Phi_T(\overline{\overline{d}}, \rho^{-1}, u, \nabla T), \tag{5}$$

$$s = \Phi_s(\overline{\overline{d}}, \rho^{-1}, u, \nabla T), \tag{6}$$

$$\overline{\overline{\pi}} = \Phi_\pi(\overline{\overline{d}}, \rho^{-1}, u, \nabla T), \tag{7}$$

$$\vec{q} = \Phi_q(\overline{\overline{d}}, \rho^{-1}, u, \nabla T). \tag{8}$$

If the Clausius–Dugem inequality is satisfied, then the determining equations (5) and (6) become the classical equations of state

$$T = \Phi_T(s, \rho^{-1}), \quad s = \Phi_s(u, \rho^{-1}),$$
$$u = \Phi_u(s, \rho^{-1}), \quad T = \Phi_T(s, \rho^{-1}) = \frac{\partial u(s, \rho^{-1})}{\partial s}. \tag{9}$$

Thus, the variables ∇T and $\overline{\overline{d}}$ drop out from the equations for the thermodynamic parameters T and s.

The static stress component is expressed in terms of Φ_u:

$$\overline{\overline{\pi}}^0 = \overline{\overline{I}} T \frac{\partial \Phi_s(u, \rho^{-1})}{\partial \rho^{-1}} = \frac{\partial \Phi_u(s, \rho^{-1})}{\partial \rho^{-1}} \overline{\overline{I}}, \tag{10}$$

where $\overline{\overline{\pi}} = \overline{\overline{\pi}}^0 + \overline{\overline{\pi}}^l$, $\overline{\overline{\pi}}^0 = -p(s, \rho^{-1})\overline{\overline{I}}$, and $p(s, \rho^{-1}) = \partial u / \partial \rho^{-1}$, $p(s, \rho^{-1})$ denoting the hydrostatic pressure. The dissipative stresses and the thermal fluxes are determined by several variables without separation.

With the principle satisfied that the properties of a material do not depend on the measuring system, we have

$$z \overline{\overline{\pi}}^l \tilde{z} = \Phi_{\pi^l}(\rho^{-1}, s, z\nabla T, z\overline{\overline{d}}\tilde{z}), \tag{11}$$

$$z\vec{q} = \Phi_q(\rho^{-1}, s, z\nabla T, z\overline{\overline{d}}\tilde{z}); \tag{12}$$

i.e., the classical separation of effects applies only to the static stress component and not to the dissipative component π^l. Furthermore, the dissipative inequality

$$\operatorname{tr}\{\overline{\overline{\pi}}^l : \overline{\overline{d}}\} + \frac{\overline{\overline{q}}}{T} \cdot \nabla T \geqslant 0 \tag{13}$$

cannot be separated into two inequalities.

Coleman and Mizel [2] have developed a procedure for separating the variables in the case of a viscous heat conducting fluid near the state $\overline{\overline{d}} = 0$ and $\nabla T = 0$.

Let us define the norm of the $\overline{\overline{d}} \oplus \nabla T$ space as follows:

$$\|\overline{\overline{d}} \oplus \nabla T\| = [\operatorname{tr}(\overline{\overline{d}}^2) + \nabla T \cdot \nabla T]^{1/2}. \tag{14}$$

Considering that the arguments of the dissipative stress component and of the thermal flux are elements of the $\|\overline{\overline{d}} \oplus \nabla T\|$ space with norm (14), and applying the mapping theorem to the functions $\overline{\overline{\pi}}^l$ and $\vec{q}$, which satisfy the objectivity principle, we arrive at the following determining equations of the first order with respect to $\overline{\overline{d}}$ for a viscous heat conducting fluid:

$$\overline{\overline{\pi}}^l = \overline{\overline{\sigma}}^l = 2\eta\overline{\overline{d}} + \lambda(\operatorname{tr}\overline{\overline{d}})\overline{\overline{I}}, \tag{15}$$

$$\vec{q} = -k\nabla T + \beta_I \overline{\overline{d}} \nabla T + \beta_{II}(\operatorname{tr} d) \cdot \nabla T \tag{16}$$

or, in the case of an incompressible fluid

$$\overline{\overline{\sigma}}^l = 2\eta\overline{\overline{d}}, \quad \vec{q} = -k\nabla T + \beta_I \overline{\overline{d}} \cdot \nabla T. \tag{17}$$

If k is always positive, moreover, then the sign of β_I depends generally on the flow pattern.

The second term in the determining equation for the thermal flux represents the effect of mechanical phenomena (strain rate tensor) on the heat transfer in the fluid. It must be emphasized here that accounting for the mechanical effect of fluid motion will yield the sought anisotropy of thermal conductivity:

TABLE 1. Thermal Conductivity k (W/m·deg) of Various Lubricants

Material	k_0, W/m·°C	$k_\perp$	$k_\parallel$	$\frac{k_\parallel}{k_0}$	$\frac{k_\perp}{k_0}$	$\frac{k_\parallel}{k_\perp}$
Solidol "s" GOST 4366-64	0,122	0,115	0,153	1,25	0,96	1,36
GOST 3276-63	0,152	0,137	0,173	1,20	0,90	1,26
Grease	0,171	0,153	0,210	1,23	0,89	1,37

$$\vec{q} = -\tilde{k}_{ij}\nabla T = -(kI - \beta_I d)\cdot\nabla T. \tag{18}$$

Thus, the thermal conductivity $\tilde{k}_{ij}$ becomes a second-rank tensor, i.e., we have a result analogous to that for oriented media [3, 4].

It is well known that a fluid constitutes an oriented flowing medium when at every point the singular orientation can be characterized by a vector n_i. The simplest theory based on the assumption of incompressibility (ρ = const), viscoelasticity ($n_i n_i = 1$), and isothermality (T = const) yields the following expressions for the orientation vector n_i, the stress tensor σ_{ij}, and the thermal flux q_i:

$$\hat{n}_i = \gamma(d_{ij}n_j - d_{km}n_k n_m n_i), \tag{19}$$

$$\sigma_{ij} = p\delta_{ij} + 2\alpha d_{ij} + (\alpha_1 + \alpha_2 d_{km}n_k n_m)\, n_i n_j + 2\alpha_3(\alpha_{jk}n_k n_i + d_{ik}n_k n_j); \tag{20}$$

$$q_i = \beta_0 T_{,h} + \beta_1 n_k n_i T_{,k}, \tag{21}$$

with the constant coefficients γ, α_i, β_i, and

$$d_{ij} = \frac{1}{2}(\dot{x}_{i,j} + \dot{x}_{j,i}); \tag{22}$$

$$\hat{n}_i = \dot{n}_i - \omega_{ij}n_j, \qquad \omega_{ij} = \frac{1}{2}(\dot{x}_{i,j} - \dot{x}_{j,i}). \tag{23}$$

From Eq. (21) we obtain

$$q_i = (\beta_0\delta_{ih} + \beta_1 n_i n_k)\frac{\partial T}{\partial x_h} = -k_{ih}\frac{\partial T}{\partial x_k}. \tag{24}$$

Here

$$-k_{ih} = k'_{ik} = \beta_0\delta_{ik} + \beta_1 n_i n_h.$$

It has been shown in [5] that an orientation may result in the process of shear flow. It is almost impossible to determine the thermal conductivity of rheological fluids during shear flow. In order to detect and measure the shear anisotropy of thermal conductivity in flowing media, therefore, it is necessary to select disperse systems with a long relaxation period. There the internal structure produced by shear flow will be retained long after the flow has ceased. Greases may serve as such a system. A method has been developed in [6] for measuring the thermal conductivity during a shear flow of Solidol, by passing this material through small orifices at a constant shearing rate of 300 sec^{-1} at a temperature of 20°C.

The results of such measurements are shown in Table 1, indicating that the thermal conductivity is higher parallel to the orientation ($k_\parallel$) than perpendicular to it ($k_\perp$). The thermal conductivity under isotropic conditions (k_0) lies inside the range $k_\perp < k_0 < k_\parallel$. Having analyzed the test data given in [6], the authors propose the following empirical formula:

$$\frac{1}{k_0} = \frac{1}{3}\left(\frac{1}{k_\parallel} + \frac{2}{k_\perp}\right). \tag{25}$$

Using the test data in Table 1 as the first approximation, and assuming that the thermal conductivity of a viscous fluid in the boundary layer around a body does not vary along the surface ($k_x = k_0$) but does vary only in the direction normal to the surface, we have for the ratio of thermal conductivities $k_y/k_x = 1.33$ approximately. For d_{xy} of the order of 150 sec^{-1}, the ratio $\beta_I d/k$ then becomes approximately equal to 1/3 ($\beta_I d/k \approx 0.33$).

We now return to relation (17) for the thermal flux density and will use it for solving the problem of convective heat transfer in an incompressible fluid in a cylindrical pipe of radius R under boundary conditions of the second kind.

Considering, for simplicity, the zone of developed steady heat transfer where the hydrodynamic profile is fully developed according to Poisseuille ($U_z = 2(1-r^2)$ for a circular pipe), we write the equation of convective heat transfer

$$v_z \frac{\partial \tilde{T}}{\partial z} = \nabla^2 \tilde{T} - K_d \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \frac{\partial \tilde{T}}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial v_z}{\partial r} \frac{\partial \tilde{T}}{\partial r} \right) \right], \tag{26}$$

where $\tilde{T}$ denotes the dimensionless temperature and where the parameter $K_d = \beta_I / 2\rho c_p R^2$ defining the effect of mechanical motion on the heat transfer will be called the critical number of dissipative heat transfer. The solution, which describes the temperature profile in a circular pipe at q_w = const, will be

$$\tilde{T} = \frac{T - T_m}{qR/k} = (1 - 4K_d)\, r^2 - \frac{r^4}{4} - \frac{7}{24} + \frac{4}{3} K_d. \tag{27}$$

It is interesting to note that, when thermal and mechanical effects interact, the Nusselt number changes by the quantity $K_d \gtrless 0$, namely

$$\mathrm{Nu} = 4.36 - K_d. \tag{28}$$

It is worthwhile to estimate the value of the K_d number, at least to the first approximation. From $\beta_I d/k \approx 0.33$ for water within the 20-50°C temperature range, we have for a pipe 2 cm in diameter (R = 1 cm) $K_d = 0.7$ approximately. For smaller pipe radii the value of K_d becomes higher. In the case of an elastic continuous medium, accounting for the state of strain will also yield an anisotropic thermal conductivity and will reveal a change in the pattern of heat transfer.

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